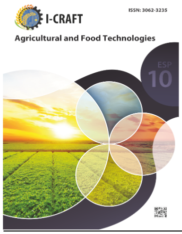

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Doi:

Algorithms for adaptive estimation of dynamic objects under the influence of additive noise

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ABSTRACT

Algorithms for adaptive evaluation of dynamic object control systems under the influence of additive noise are considered. Algorithms of the Kalman filter are given. An adaptive filtering approach for nonlinear systems with additive noise is also considered. The developed adaptive estimation algorithm can calculate the square root of the covariance matrix in a simple way in such a way that positive semi-certainty is guaranteed, which significantly increases the stability and accuracy of the filter.

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1. Introduction

The Kalman filter is widely used in numerous tasks of synthesizing and designing systems for managing dynamic objects of various functional purposes [1-9]. The Kalman filter provides an unbiased estimate with minimal variance about the state of a discrete linearly varying dynamic system, the input and output of which are distorted by Gaussian white noise with an additive character. This approach was extended to continuous dynamical systems by Kalman and Bucy with a linear character [2,4].

The Kalman filter has one major drawback [3,4,10]. The equations used in the optimal filter require precise knowledge of the dynamic equations of the system and the statistics of random variables, including the need to know the transition matrices of the system and the covariance of disturbances such as additive white noise. However, usually only their estimates are available. Recently, Kalman filter schemes [1,3,4,11] have appeared in order to circumvent this problem. These schemes are commonly referred to as "adaptive filters". Various adaptive filters can be grouped according to the principle of identifying undefined parameters, heuristic weighting coefficients, or the absence of correlation of residual terms.

Theoretically and practically, the synthesis of control systems for dynamic objects very often addresses estimation issues in which measurement uncertainty is represented as an additive purely random sequence or white noise. At the same time, there are estimation problems [2,3,4,10,12] when an additive Markov sequence, i.e. sequentially correlated or non-white noise, is a more accurate model of uncertainty in measurements.

When using systems used to control dynamic objects, the structural and parametric data of the controller do not have a dependence associated with the structure and parameters of the observer. This, in turn, makes it possible to use well-known control laws, and in the future, the possibility of adapting them using the evaluation contour. It should be borne in mind that the use of the given structure will be convenient in the event that there is a need to modernize or adapt the existing management system. When studying the evaluation algorithm with relatively high performance, the qualitative indicators of the adaptive system are not much inferior to the non-adaptive system in terms of minimum indicators. But even at the same time, during the period of transients carried out in the observer, the qualitative indicators of a closed system may have deteriorations reaching the loss of asymptotic stability. Such disadvantages can be eliminated by increasing the speed of estimation algorithms in areas with large deviations.

In general, considering the formulation of the problem, multiple observations may contain certain white noises, while non-white Markov-type noises may not contain noise as a whole or be considered as some combination of the three studied possibilities. Following from this, in further descriptions, under the terminology "non-white noise" we will understand the presence of Markov-type noise or the non-participation of noise as a whole in one or more dimensions

2. Materials and Methods

Consider a linear dynamical system described by the equation

$$x_{i+1} = A_{i+1|i}x_i + \Gamma_{i+1|i}w_i, \quad \textbf{(1)}$$

$$z_{i+1} = H_{i+1}x_{i+1} + v_{i+1} \quad \textbf{(2)}$$

for $i = 0,1,2,...$ with an initial condition x_0 and a measurement matrix H_{i+1}.

The measurement error v_{i+1} is identified with the state vector of some additional linear dynamic system (forming filter) with a transition matrix Ψ_i and a perturbation vector ξ_i: $v_{i+1} = \Psi_i v_i + \xi_i$ for $i = 1,2,...$ with an initial condition v_0.

It is assumed that perturbations $\{w_i, i = 0,1,2,... \}$ and $\{\xi_i, i = 0,1,2,... \}$ – are sequences of random vectors with known correlation matrices $E[w_i w_i^T] = Q_i, E[\xi_i \xi_i^T] = R_i$, where E – is the averaging operator and "T" is the transposition operation. These two successive equations do not depend on each other and also do not depend on the initial conditions x_0, v_0.

With mutually uncorrelated errors $\{v_i, i = 0,1,2,... \}$ $E[v_i v_i^T] = V_i \delta_{ki}$, where δ_{ki} – is the symbol called Kronecker, is optimal in the form of a minimum of the variance estimate $\hat{x}_{i|i}$ of the state vector x_i of the system (1), based on measurements $\{z_k, k = 1,2,..., i \}$ of the form (2), it is formed according to the recurrent Kalman algorithm [1-11]:

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + K_i\left(z_i - H_i\hat{x}_{i|i-1}\right) \quad \textbf{(3)}$$

$$\hat{x}_{i|i-1} = A_{i|i-1}\hat{x}_{i-1|i-1} \quad \textbf{(4)}$$

ISSN: 3062-3235
I-CRAFT AGRICULTURAL and FOOD TECHNOLOGIES
Doi:

$$K_i = P_{i|i-1}H_i^T(H_iP_{i|i-1}H_i^T + V_i)^{-1} \quad (5)$$

$$P_{i|i-1} = A_{i|i-1}P_{i-1|i-1}A_{i|i-1}^T + \Gamma_{i|i-1}Q_{i-1}\Gamma_{i|i-1}^T \quad (6)$$

$$P_{i|i} = (I - K_iH_i)P_{i|i-1} \quad (7)$$

for $i = 1,2,...$, where I – is a unit matrix, and the initial conditions for equations (4) and (6) are set, respectively, by an a priori estimate $\hat{x}_{0|0}$ of the initial state vector x_0 and the correlation matrix $P_{0|0}$ of its error $\tilde{x}_{0|0} = \hat{x}_{0|0} - x_0$, uncorrelated with $\{w_i, \xi_i, i = 1,2, ... \}$. In this case, $P_{i|i}$ is a correlation matrix of the error of the optimal estimate $\hat{x}_{i|i}$ of the current state x_i, calculated using formulas (5) – (7) without using measurements z_i.

To solve the estimation problem under the influence of additive noise, the following algorithms can be proposed [2-4,13-14]. It is assumed that the statistical property of system noise is known. However, in real time, the process noise covariance matrix Q or the observation noise variance matrix R are often unknown. In addition, these parameters may change over time. Therefore, an adaptive Kalman filter must be designed to adjust Q and R, where it is important to increase the accuracy and stability of filtration.

Thus, we assume an adaptive filtering algorithm based on the maximum a posteriori estimate [13-15]. Subsequently, the algorithm has the ability to evaluate unknown time-varying noise. The specific calculation process is as follows:

$\hat{x}_0 = E[x_0], P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$,

$\hat{Q}_0 = Q(0), \hat{R}_0 = R(0)$ during initialization $i = 0$.

$\hat{x}_{i|i-1} = A_{i|i-1}\hat{x}_{i-1|i-1}, P_{i|i-1} = A_{i|i-1}P_{i-1|i-1}A_{i|i-1}^T - \hat{Q}_{i-1}$

during iteration $i = 1,2,...$, that is, the time update.

Let's evaluate the statistical properties of measurement noise when updating measurement results:

$$\tilde{z}_i = z_i - H_i\hat{x}_{i|i-1}, \quad (8)$$

$$\hat{R}_i = (1 - d_i)\hat{R}_{i-1} + d_i(\tilde{z}_i\tilde{z}_i^T - H_iP_{i|i-1}H_i^T). \quad (9)$$

Let's estimate the value of the state, after correction we will calculate the a posteriori variance of the state:

$$K_i = P_{i|i-1}H_i^T(H_iP_{i|i-1}H_i^T + \hat{R}_i)^{-1}, \quad (10)$$

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + K_i\tilde{z}_i, \quad (11)$$

$$P_i = (I - K_iH_i)P_{i|i-1}. \quad (12)$$

Then we will evaluate the statistical properties of the process noise in accordance with

$$\hat{Q}_i = (1-d_i)\hat{Q}_{i-1} + d_i(K_i\tilde{z}_i\tilde{z}_i^TK_i^T + P_{i|i} - A_{i|i-1}P_{i-1|i-1}A_{i|i-1}^T),$$

where $d_i = (1 - b)/(1 - b^{i+1})$, b it is a factor of forgetting, and $0 < b < 1$.

Now let's consider the adaptive filtering approach for nonlinear systems with additive noise. Both the process equations and the measurement equations are nonlinear according to

$$x_i = f(x_{i-1}) + w_{i-1}, \quad (13)$$

$$z_i = h(x_i) + v_i, \quad (14)$$

where $f(\square)$ and $h(\square)$ they are non-linear functions

3. Results and Discussion

The adaptive assessment algorithm is as follows:

$$\hat{x}_0 = E[x_0], \; S_0 = chol\left\{E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]\right\}, \quad (15)$$

$$\sqrt{\hat{Q}_0} = S_0, \sqrt{\hat{R}_0} = chol\left\{E[(z_0 - \hat{z}_0)(x_0 - \hat{z}_0)^T]\right\}. \quad (16)$$

$$\chi_{i-1} = [\hat{x}_{i-1}\hat{x}_{i-1} + \gamma S_{i-1}\hat{x}_{i-1} - \gamma S_{i-1}], \text{при } i = 1,2,...,\infty. \quad (17)$$

Let's write the time update equations as follows:

$$\hat{\chi}_i^* = f(\chi_{i-1}), \quad (18)$$

Doi:

$$\hat{x}_{i|i-1} = \sum_{k=0}^{2L} W_k^{(m)} \hat{\chi}_{i,k}^{*}, \tag{19}$$

$$S_{i|i-1} = qr\left\{[\sqrt{W_1^{(c)}}(\hat{\chi}_{i,1:2L}^{*} - \hat{x}_{i|i-1})\sqrt{\hat{Q}_{i-1}}]\right\}, \tag{20}$$

$$S_{i|i-1} = cholupdate\left\{[S_{i|i-1}, \hat{\chi}_{i,0}^{*} - \hat{x}_{i|i-1}, W_0^{(c)}]\right\}. \tag{21}$$

Then we calculate the square root of the measured noise matrix:

$$\hat{Z}_{i-1}^{*} = h(\hat{\chi}_{i-1}), \tag{22}$$

$$\hat{z}_{i-1}^{*} = \sum_{k=0}^{2L} W_k^{(m)} \hat{Z}_{i-1,k}^{*}, \tag{23}$$

$$\tilde{z}_{i-1}^{*} = z_{i-1} - \hat{z}_{i-1}^{*} \tag{24}$$

$$\sqrt{R^{**}} = cholupdate\left\{\sqrt{1-d_i}\sqrt{\hat{R}_{i-1}}, |\tilde{z}_{i-1}^{*}|, d_i\right\}, \tag{25}$$

$$\sqrt{R^{*}} = cholupdate\left\{\sqrt{R^{**}}\hat{Z}_{i-1|0:2L}^{*} - \hat{z}_{i-1}^{*}, -d_i W_k^{(c)}\right\}, \tag{26}$$

$$\sqrt{\hat{R}_i} = diag\left\{\sqrt{R^{*}}\right\} \tag{27}$$

where;

$$\hat{\chi}_{i|i-1} = [\hat{x}_{i|i-1}\hat{x}_{i|i-1} + \gamma S_{i|i-1}\hat{x}_{i|i-1} - \gamma S_{i|i-1}], \tag{28}$$

$$\hat{Z}_{i|i-1} = h(\hat{\chi}_{i|i-1}), \tag{29}$$

$$\hat{z}_{i|i-1} = \sum_{k=0}^{2L} W_k^{(m)} \hat{Z}_{i|i-1,k}, \tag{30}$$

$$\tilde{z}_{i-1}^{*} = z_{i-1} - \hat{z}_{i|i-1}. \tag{31}$$

The measurement update equations are as follows:

$$P_{(xz)i} = \sum_{k=0}^{2L} W_k^{(c)} (\hat{\chi}_{i|i-1,k} - \hat{x}_{i|i-1})(\hat{Z}_{i|i-1,k} - \hat{z}_{i|i-1})^T, \tag{32}$$

$$S_{(z)i} = qr\left\{\left[\sqrt{W_1^{(c)}}(\hat{Z}_{i|i-1,1:2L} - \hat{z}_{i|i-1})\sqrt{\hat{R}_i}\right]\right\}, \tag{33}$$

$$S_{(z)i} = cholupdate\left\{[S_{(z)i}, \hat{Z}_{i|i-1,0} - \hat{z}_{i|i-1}, W_0^{(c)}]\right\} \tag{34}$$

$$K_i = \frac{\left(P_{(xz)i} / S_{(z)i}^T\right)}{S_{(z)i}}, \tag{35}$$

$$\hat{x}_i = \hat{x}_{i|i-1} + K_i \tilde{z}_i \tag{36}$$

Doi:

$$U = K_i S_{(z)i}, \tag{37}$$

$$S_{(z)i} = cholupdate\{S_{i|i-1}, U, -1\}. \tag{38}$$

Let's estimate the square root of the process noise matrix in accordance with;

$$\sqrt{Q^{**}} = cholupdate\left\{\sqrt{\hat{Q}_{i-1}}, |\hat{x}_i - \hat{x}_{i|i-1}|, d_i\right\} \tag{39}$$

$$\sqrt{Q^{*}} = cholupdate\left\{\sqrt{Q^{**}}, U, -d_i\right\} \tag{46}$$

$$\sqrt{\hat{Q}_i} = diag\left\{diag\left(\sqrt{Q^{*}}\right)\right\}, \tag{41}$$

where $d_i = (1-b)/(1-b^{i+1})$ and b it is a factor of forgetting, as a rule $0 < b < 1$.

The weights ($W_k^{(m)}$ and $W_k^{(c)}$) of the mean value and covariance are given by the formula;

$$W_0^{(m)} = \frac{\lambda}{L+\lambda}, \tag{42}$$

$$W_0^{(c)} = \frac{\lambda}{L+\lambda} + 1 - \alpha^2 + \beta, \tag{43}$$

$$W_k^{(m)} = W_k^{(c)} = \frac{1}{2(L+\lambda)},\ k = 1,\ldots,2L, \tag{44}$$

where $\lambda = \alpha^2(L+\kappa)$ it is a scaling parameter. The constant α defines the spread of sigma points around the average value, which is usually set to a small positive value (for example, $10^{-4} \le \alpha \le 1$). Constant $\kappa \ge 0$ is a secondary scaling parameter. $\beta \ge 0$is used to account for prior knowledge of the distribution (for Gaussian distributions, the optimal value is $\beta = 2$) [4-10]. In addition, $\gamma = \sqrt{L+\lambda}$. Here we used three powerful linear algebra methods: QR decomposition ($qr\{\}$), updating the Xoleski coefficient ($cholupdate\{\}$) and efficient least squares methods (/), which are briefly discussed in [13,14,16

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